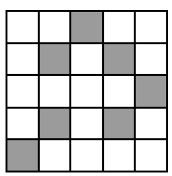
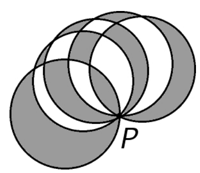
Answer Key	
1. \$45.20	4. $\frac{7}{25}$
2. 7	5. 720
3. 16	6. $\frac{3}{5}$
	7. $1/(-x+2)$

1. According to the information we are given, the price increases at a rate of \$7.40 every ten seconds, or \$0.74 per second. Since 45 seconds elapse between 8:48:20 and 8:49:05, an additional (45)(\$0.74) = \$33.30 is added to the bill after the display reads \$11.90, for a total of **\$45.20**.

2. Each row must contain at least one shaded square, but adjacent rows have different numbers of shaded squares, which means that there will need to be at least two more shaded squares, in the second and fourth row. The question is whether seven shaded squares can be arranged so that the columns also satisfy the given conditions. As the diagram demonstrates, this can be done, so **7** shaded squares suffice.



3. We place the circles relatively close together, as shown, in order to understand what happens as we draw each successive circle. For example, the fifth circle cuts through each of the first four, creating five new regions. In total we have $(1+2+3+4+5)+1 =$ **16** regions, including the region surrounding all the circles. (Note that the answer can be obtained with nothing more than an accurate sketch and some careful counting.)



4. Although it is usually not advantageous to create fractions when solving an equation, we will make an exception here and divide through by 5 before squaring both sides. The result is

$$(\sqrt{1+x})^2 + 2(\sqrt{1+x})(\sqrt{1-x}) + (\sqrt{1-x})^2 = \left(\frac{7\sqrt{2}}{5}\right)^2.$$

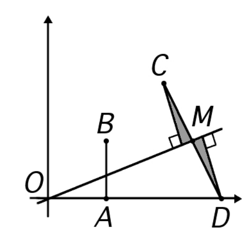
Simplifying, we obtain

$$\begin{aligned} & (1+x) + 2\sqrt{1-x^2} + (1-x) = \frac{98}{25} \\ \Rightarrow \quad & 2 + 2\sqrt{1-x^2} = \frac{98}{25} \\ \Rightarrow \quad & \sqrt{1-x^2} = \frac{24}{25} \\ \Rightarrow \quad & x^2 = 1 - \frac{576}{625} = \frac{49}{625}. \end{aligned}$$

Therefore the positive solution to this equation is **7/25**.

5. Suppose that Austin currently possesses x shirts, y pairs of pants, and z pairs of shoes. If he were to obtain one more shirt, then he would gain yz new outfits, since each would involve the new shirt, one of y pairs of pants, and one of z pairs of shoes. Similarly, buying another pair of pants leads to xz additional outfits, and more shoes gives xy extra outfits. We know that $yz = 48$, $xz = 90$, and $xy = 120$. Multiplying these and taking the square root gives $\sqrt{x^2y^2z^2} = \sqrt{(48)(90)(120)}$, which reduces to $xyz = 720$. In other words, Austin currently can create **720** outfits.

6. The key to solving the problem lies in determining the line through O equidistant from C and D. It turns out that this line is the one passing through the midpoint M of segment $\overline{CD}$. Note that $\overline{CM}$ and $\overline{DM}$ are not themselves perpendicular to OM. However, if we draw the segments that are perpendicular, creating the slim shaded right triangles, then the resulting triangles are congruent. (Proof?) This shows that line OM is the same distance from C and D. Hence any point P on $\overline{AB}$ above the intersection with OM will result in a line that is closer to C than to D. We find that M has coordinates $(1, \frac{5}{2})$, so line OM has equation $y = \frac{2}{5}x$, which intersects the line $x = 1$ at the point $(1, \frac{2}{5})$. Thus P can be located anywhere on the upper portion of $\overline{AB}$, which has length **3/5**.



7. One means for deducing the correct function $g(x)$ is to observe that if a and b are any two consecutive terms in the sequence, then $a + 1/b = 2$.

(Try this with a few pairs of terms to see how this works.) Solving for b leads to $b = 1/(2-a)$. Therefore the desired function is given by $g(x) = 1/(2-x)$, or $g(x) = 1/(-x+2)$. (Either answer is fine.)

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⋆ Regional Level ⋆

The Mandelbrot Competition

Round Two Solutions

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